Form B Solutions

Clover Math

April 2024

1. The first 5 odd numbers in question are 1, 3, 5, 7, and 9. The sum is $1+3+5+7+9 = \boxed{25}$. Observe that this is 5^2. Does this pattern hold? Why?
2. William is $4*12+3 = 51$ inches tall and his brother is $5*12+2 = 62$ inches tall. William's brother is taller than him by $62-51 = \boxed{11 \text{ inches}}$.
3. There are 10 even numbers less than or equal to 20, and of these, the numbers 6, 12, and 18 are divisible by six. Therefore, we have $10-3 = \boxed{7}$ numbers that work.
4. Three in five clovers will be flowers, so the total number of flowers is $\frac{3}{5} \cdot 35 = \boxed{21}$.
5. There are 3 numbers that she could roll that are less than 4, and her die can roll 6 numbers. Therefore, the probability is $\frac{3}{6} = \boxed{\frac{1}{2}}$.
6. There are $\frac{2}{3}$ of the mathematicians, but double the time, so our answer is $3 \cdot \frac{2}{3} \cdot 2 = \boxed{4}$.
7. For the folds to be clean, each of the folded pieces must be equal in area to the parts of the paper they are folded over. Since we only deal with the areas of the pieces that are folded over when we find the new area, we are left with $\frac{1}{2}$ of the original area, so our ratio is $\boxed{2}$.
8. For convenience, we can use $e \approx 2.72$ instead of the given decimal. $5*2.72 = 13.6$ which rounds up to $\boxed{14}$.
9. We simply plug in $a = 5$ and $b = 3$ into our function.
$$5\$3 = 5*3 - \frac{5}{3} = \frac{45-5}{3} = \boxed{\frac{40}{3}}$$
10. Let's say Brian's height is x, Arthur's height is y, and Andrew's height is z. Then, we can express the given information as $y = \frac{5}{4}x$ and $z = \frac{6}{5}y$. Then,
$$z = \frac{6}{5} * \frac{5}{4}x = \frac{6}{4}x = \frac{3}{2}x,$$
so our answer is $\boxed{50\%}$.
11. Notice that $20 = \frac{5}{3} * 12$, so we can simply multiply the value of $12x + 12$ to get our answer.
$$20x + 20 = \frac{5}{3}(12x+12) = \frac{5}{3} * 30 = \boxed{50}$$
12. The given information tells us that $DE = 16$ and $EC = 4$. The area of ADE is
$$\frac{1}{2}AB * DE = \frac{1}{2} * 20 * 16 = 160.$$
Now, notice that triangles ADE and FCE are similar with side length ratios of $4:1$. This means that the area of FCE is $\frac{1}{16}$ of the area of ADE, or $\frac{1}{16} * 160 = \boxed{10}$.

13. We want the least common multiple of 36 and 15, which is 180. 180 minutes translates into 3 hours and three hours after 9 A.M. is $\boxed{\text{12:00 P.M.}}$

14. The 4 side faces are unaffected by the changes we've made. However, now we only have one of the 4x5 sides, and we've also cut $\frac{1}{4}$ of its area out of it, so it only has an area of 15 now. Therefore, our surface area is

$$15 + 2(4 * 6) + 2(5 * 6) = \boxed{123}$$

15. Currently, we have $2 * 0.75 = 1.5$ gallons of juice. For the final solution to be half-juice, we need 3 total gallons of solution. Therefore, we need to add $3 - 2 = \boxed{1}$ gallon of water to our current solution.

16. Let the second largest of the three integers be x. Note that the average of the three integers is x, so $x = \frac{777}{3} = \boxed{259}$.

17. Note that the number of strawberries, since it leaves the same remainder when divided by both 5 and 7, must be that remainder divided by a common multiple of 5 and 7. Since it can't be zero, we use the LCM of 35. There are $35 + 3 = \boxed{38}$ strawberries.

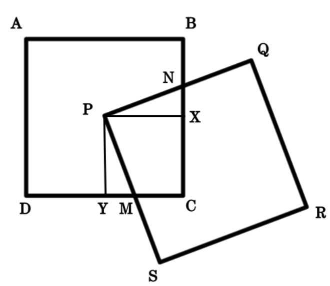
18. The numerator is an arithmetic sequence with common difference 1 and the denominator is an arithmetic sequence with common difference 3. We can represent this using algebra: the nth term of the sequence is $\frac{39+n}{12+3n}$. Then, we set up an equation using our desired condition and solve:

$$2(39 + n) = 12 + 3n \rightarrow n = 2 * 39 - 12 = \boxed{66}$$

19. We'll deal with powers of 6 first. $\frac{96}{6} = 16$, and 16 is not divisible by 6. Therefore, we only have one power of 6 and $b = 1$. We know that $16 = 4^2$, which gives us $a = 1$. Our answer is $a + b = \boxed{3}$.

20. The smallest prime is 2, so if we can obtain it, it is our answer. Trying 11 and 22 fail, but trying a sum of 33 gives us 2 and 31, which works. Our answer is $\boxed{2}$.

21. Trying to directly solve the area of the quadrilateral might seem a little bit daunting. Let's draw some altitudes from P to BC and CD.



Notice that triangles PYM and PXN are congruent. If you simply switched them, you would get the square $PXCY$, which has the same area as $PMCN$. Square $ABCD$ has side length 10 and area 100, and $PXCY$ has one-fourth the area of $ABCD$. The answer is thus $\frac{1}{4} * 100 = \boxed{25}$.

22. We do casework by the different digits that 1 can be located in.

1 shows up once in the units digit ein $1, 11, 21, ..., 101$, which is 11 times.

1 shows up in the tens digit in the numbers $10, 11, 12, ..., 19$, which is 10 times.

1 shows up in the hundreds digit of all the 3-digit numbers, $100, 101, 102, 103, 104, 105$, which is 6 times.

Adding these up, we find that 1 shows up $11 + 10 + 6 = \boxed{27}$ times.

23. Only the units digit's powers matter, so the question would be the same if we asked for 4^{2024}. Trying out the first few powers of four, we get units digits of 4 for 4^1, 6 for 4^2, 4 for 4^3, and so on in a repeating pattern. Even powers give us a units digit of 6, and 2024 is even, so the answer is $\boxed{6}$.

24. The units digit must be even, so it can be 2, 4, or 6. Since the digits of the number have to add up to a multiple of 3, we can have a tens digit of 1 or 4 when the units digit is 2; a tens digit of 2 or 5 when the units digit is 4, and a tens digit of 3 or 6 when the units digit is 6; giving us 6 working numbers. There are 36 total numbers we can form by rolling the two dice, so our probability is $\frac{6}{36} = \boxed{\frac{1}{6}}$.

25. We will get a 100-digit number after this subtraction. For this number, the units digit is 6, the tens digit is 6, the hundreds digit is 7, the thousands digit is 8, and all the other 96 digits are 9. The sum of these digits is $96 * 9 + 8 + 7 + 6 + 6 = \boxed{891}$.

26. Firstly, $n > 6$ since otherwise 16_n would be undefined. Then, its guessing and checking until you find a working n. $n = 7$ does not work since $21_7 = 15$, which is not prime. Trying $n = 11$, we see that $21_{11} = 23$, which is prime, and $16_{11} = 17$, which is also prime. Seems like $n = \boxed{11}$ is our answer.

27. First, treat the four students as a single entity. The teacher has 5 entities to order, which gives $5! = 120$ ways. However, we still need to consider the $4! = 24$ ways the group of four can order themselves, so our answer is $24 * 120 = \boxed{2880}$.

28. The 7th triangular number is

$$1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$$

and the 8th triangular number is

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36,$$

so the sum of the two is $28 + 36 = \boxed{64}$. In fact, the sum of the $n - 1$th triangular number and the nth triangle number is n^2 (can you prove why?).

29. Notice that $\frac{425}{123}$ is between 3 and 4; this tells us that $a = 3$ since the other part of the expression is less than one. We subtract this out and apply similar logic to get b:

$$\frac{1}{b + \frac{1}{c + \frac{1}{d}}} = \frac{425}{123} - 3 = \frac{56}{123} \to b + \frac{1}{c + \frac{1}{d}} = \frac{123}{56} \to b = 2$$

We do this again for c:

$$\frac{1}{c + \frac{1}{d}} = \frac{123}{56} - 2 = \frac{11}{56} \to c + \frac{1}{d} = \frac{56}{11} \to c = 5$$

And finally, for d:

$$\frac{1}{d} = \frac{56}{11} - 5 = \frac{1}{11} \to d = 11$$

Our answer is therefore $3 + 2 + 5 + 11 = \boxed{21}$.

30. We have two cases for this problem: the first card is an Ace of Spades or the first card is an Ace of one of the other 3 suits.

If the first card is an Ace of Spades, there will be 12 other Spades for the second card, which will produce 12 possible selections.

If the first card is not an Ace of Spades, we can pick between the other 3 Aces for the first card and the 13 Spades for the second card, giving us $3 * 13 = 39$ possible selections.

We have $12 + 39 = 51$ total selections that work, with $52 * 51$ selections being possible. Our probability is $\frac{51}{52*51} = \boxed{\frac{1}{52}}$.